

CJ-Drawdown+Recovery–A workbook for Cooper-Jacob analysis of drawdown and recovery

Single-well aquifer tests frequently are analyzed with Cooper-Jacob ([1946](#)) and Theis recovery ([Jacob, 1963](#)) methods because analyses are simple. Transmissivity is estimated by fitting a straight line to drawdowns or residual drawdowns on an arithmetic axis vs. time on a logarithmic axis in a semi-log plot. Transmissivities are inversely proportional to slopes of lines in both methods. Drawdowns and residual drawdowns in confined and unconfined aquifers have been analyzed by many practitioners using both methods, regardless of differences between field conditions and theory. Estimates from these constant rate tests frequently represent aquifer transmissivities despite mismatches between field conditions and theory ([Halford and others, 2006](#)).

Slopes of drawdowns and residual drawdowns are identical, but plotted data during drawdown and recovery are displaced (Figure 1). Drawdowns are displaced along the Y-axis because well losses increase overall drawdown during pumping. This displacement generally is uniform because flow rate should be constant while pumping. Residual drawdowns are plotted on a transformed X-axis of time-since-pumping-started divided by time-since-pumping-stopped, $\Delta t-Q_{start}/\Delta t-Q_{stop}$ (Jacob, 1963). Increasing time plots to the left with transformed recovery data, where $\Delta t-Q_{start}/\Delta t-Q_{stop}$ have large values when recovery starts and theoretically diminish to 1 after infinite recovery (Figure 1). Plotted positions of drawdown and residual drawdown data are largely irrelevant because estimated slopes determine transmissivity estimates.

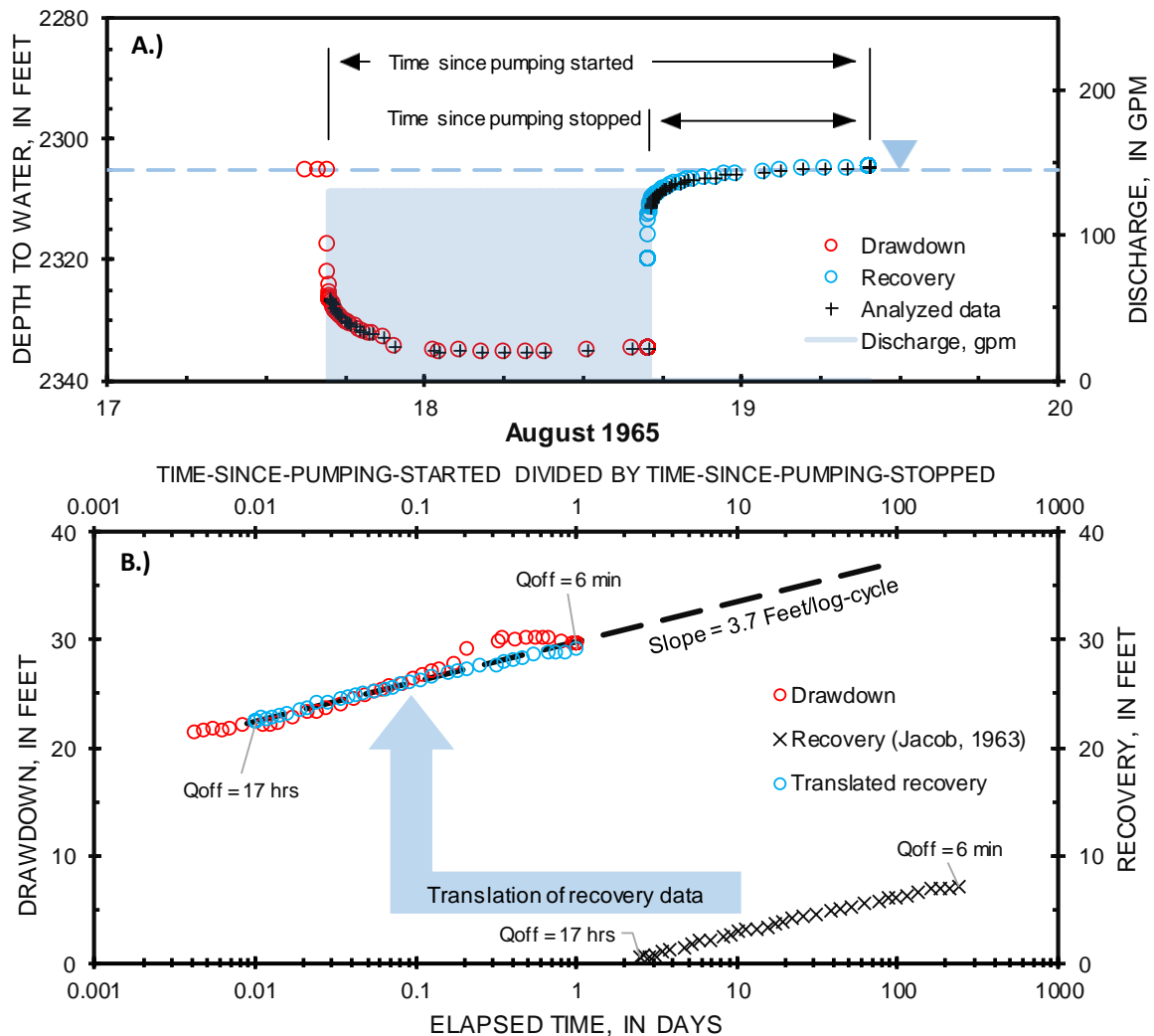


Figure 1.— Examples of pumping test results in well [UE-19fS \(2565-4779 ft\)](#) from August 1965. A, Cartesian plot of changes in depth to water relative to drawdown and recovery; B, Semi-log plot of translated recovery data combined interpretation of drawdown and recovery.

Residual drawdowns can be displaced to coincide with drawdowns so that drawdown and recovery could be interpreted simultaneously with a single line (Figure 1). Translated recovery data is shifted left on the X-axis of $\Delta t-Q_{start}/\Delta t-Q_{stop}$ so that the first recovery observation coincides with the last drawdown observation. Translated recovery data is shifted up on the Y-axis by the difference between averages of drawdowns and residual drawdowns (Figure 2). Translation does not affect the slope so drawdown and recovery data can be interpreted simultaneously. This approach is a refinement of previously developed workbooks ([Halford and Kuniansky, 2002](#); [Frus and Halford, 2018](#)).

Simultaneous interpretation of Cooper-Jacob ([1946](#)) and Theis recovery ([Jacob, 1963](#)) methods has been implemented in the workbook CJ-Drawdown+Recovery-2019.xlsm (Figure 2). A continuous series of antecedent, pumping, and recovery water levels in the pumped well are specified as depth to water or water level above the transducer. Analyzable drawdown and residual drawdowns are plotted on a semi-log plot. A straight line initially is regressed to drawdown and recovery data with the “GROSS FIT” button. Fit between straight line and plotted data can be refined visually to ignore outliers with the “ADJUST LINE” button. Transmissivity is reported with a user defined number of significant digits. Average hydraulic conductivity is reported if an assumed thickness is specified.

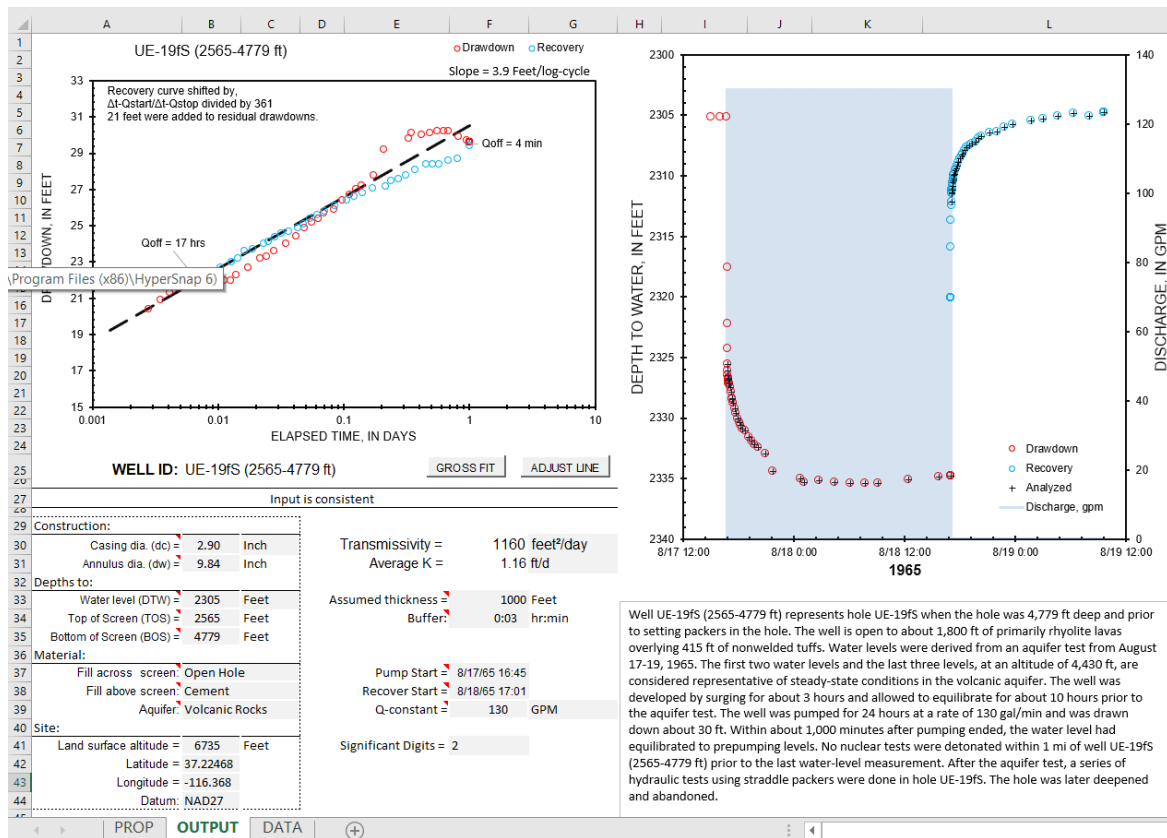


Figure 2.— OUTPUT page in CJ-Drawdown+Recovery-2019.xlsm with data from well [UE-19fS \(2565-4779 ft\)](#) from Frus and Halford ([2018](#)).

CJ-Drawdown+Recovery-2019.xlsm and explanatory PDF can be downloaded with the following link.

References

- Cooper, H.H., and C.E. Jacob. 1946. A generalized graphical method for evaluating formation constants and summarizing well field history. American Geophysical Union Transactions 27: 526–534. <https://doi.org/10.1029/TR027i004p00526>
- Frus, R.J., and Halford, K.J., 2018, Documentation of single-well aquifer tests and integrated borehole analyses, Pahute Mesa and Vicinity, Nevada: U.S. Geological Survey Scientific Investigations Report 2018–5096, 22 p., <https://doi.org/10.3133/sir20185096>.
- Halford, K.J. and E.L. Kuniansky 2002, Documentation of spreadsheets for the analysis of aquifer pumping and slug test data, USGS OF 02-197
<https://pubs.usgs.gov/of/2002/ofr02197/>
- Halford, K.J., Weight, W.D., and Schreiber, R.P., 2006, Interpretation of transmissivity estimates from single-well pumping aquifer tests: Groundwater, v. 44, no. 3, p. 467–471, <https://doi.org/10.1111/j.1745-6584.2005.00151.x>.
- Jacob, C.E., 1963, The recovery method for determining the coefficient of transmissibility, in Bentall, R., ed., Methods of determining permeability, transmissibility, and drawdown: U.S. Geological Survey Water-Supply Paper 1536–I, p. 283–292, <https://pubs.er.usgs.gov/publication/wsp1536I>.

CJ-Drawdown+Recovery-2019.xlsm Workbook

The workbook consists of three pages, DATA, OUTPUT, and PROP, and one hidden page, CONTROL. The hidden CONTROL page contains lookup tables and code for translating coordinates, which users should not need to edit. Aquifer test information, analysis, and results are summarized on the OUTPUT page. Well construction, aquifer thickness, aquifer material, site identifier, and remarks about the test are specified on the OUTPUT page. Additional information such as a well construction diagram and pictures of the site also could be pasted on the OUTPUT page. Time series of water-level changes from data loggers or manual measurements are entered on the DATA page. Ranges of transmissivities for hydrogeologic units are specified on the PROP page. The list should be modified to include more specific information about local hydrogeologic units in a study area.

DATA page

A continuous series of antecedent, pumping, and recovery water levels in the pumped well are specified as depth to water or water level above the transducer in columns A-B from row 6 down (Figure 3). Time can be specified as decimal days, which is the convention in Excel, or as elapsed time in seconds, minutes, or hours as specified in cell A5. Units of measured water levels are specified in cell B5. An initial date and time (cell D3) are specified only if water levels are paired with elapsed times. Units of analyzed water-level changes can differ as specified in cell E5.

| | A | B | C | D | E | F | G | H | I | J |
|---|---------------------------|--------|---|------------------|----------------|-------|---|---|---|---|
| 1 | | | | | | | | | | |
| 2 | INPUT | | | Date and time | Depth to water | | | | | |
| 3 | Overwrite with your data. | | | Initial: | 2,305.20 | C | t | | | |
| 4 | ↓ | ↓ | | | | | | | | |
| 5 | Date-Time | Feet | | Date-Time | Feet | [[[Cy | | | | |
| 6 | 08/17/1965 15:00 | 2305.3 | | 08/17/1965 15:00 | 2,305.20 | | | | | |
| 7 | 08/17/1965 16:00 | 2305.3 | | 08/17/1965 16:00 | 2,305.20 | | | | | |
| 8 | 08/17/1965 16:45 | 2305.3 | | 08/17/1965 16:45 | 2,305.20 | | | | | |
| 9 | 08/17/1965 16:46 | 2317.7 | | 08/17/1965 16:46 | 2,317.60 | | | | | |
| 10 | 08/17/1965 16:47 | 2322.3 | | 08/17/1965 16:47 | 2,322.20 | | | | | |
| 11 | 08/17/1965 16:48 | 2324.4 | | 08/17/1965 16:48 | 2,324.30 | | | | | |
| 12 | 08/17/1965 16:49 | 2325.7 | | 08/17/1965 16:49 | 2,325.60 | | | | | |
| <div> ◀ ▶ Defaults OUTPUT DATA ⊕ </div> | | | | | | | | | | |

Figure 3.—DATA page in the CJ-Drawdown+Recovery-2019 workbook where time series of water levels are specified.

Data Page

Clear existing data between columns A and B from row 6 to the last entry.

| | A | B | C |
|----|---------------------------|--------|------|
| 1 | | | |
| 2 | INPUT | | |
| 3 | Overwrite with your data. | | |
| 4 | | | |
| 5 | Date-Time | Feet | |
| 6 | 08/17/1965 15:00 | 2305.3 | |
| 7 | 08/17/1965 16:00 | 2305.3 | |
| 8 | 08/17/1965 16:45 | 2305.3 | |
| 9 | 08/17/1965 16:46 | 2317.7 | |
| 10 | 08/17/1965 16:47 | 2322.3 | |
| | Defaults | OUTPUT | DATA |

Empty cells before adding your data.

| | A | B | C |
|----|---------------------------|--------|------|
| 1 | | | |
| 2 | INPUT | | |
| 3 | Overwrite with your data. | | |
| 4 | | | |
| 5 | Date-Time | Feet | |
| 6 | | | |
| 7 | | | |
| 8 | | | |
| 9 | | | |
| 10 | | | |
| | Defaults | OUTPUT | DATA |

Specify units for time in cell A5.

| | A | B | C |
|----|------------------|--------|---|
| 5 | Date-Time | Feet | |
| 6 | Second | 2305.3 | |
| 7 | Minute | 2305.3 | |
| 8 | Hour | 2305.3 | |
| 9 | Day | 2305.3 | |
| 10 | Date-Time | 2317.7 | |
| | 08/17/1965 16:47 | 2322.3 | |

Specify units for measured water levels in cell B5.

| | A | B | C |
|----|------------------|--------|---|
| 5 | Date-Time | Feet | |
| 6 | 08/17/1965 | Inch | |
| 7 | 08/17/1965 | Feet | |
| 8 | 08/17/1965 | Meters | |
| 9 | 08/17/1965 | cm | |
| 10 | 08/17/1965 | mm | |
| | 08/17/1965 16:47 | 2322.3 | |

Specify units for depths to water that are analyzed in cell E5.

| | D | E | F | G | H | I |
|----|------------------|----------------|----|---|---|---|
| 2 | Date and time | Depth to water | | | | |
| 3 | | 2,305.20 | ft | | | |
| 4 | | | | | | |
| 5 | Date-Time | Feet | | | | |
| 6 | 08/17/1965 15:00 | Inch | | | | |
| 7 | 08/17/1965 16:00 | Feet | | | | |
| 8 | 08/17/1965 16:45 | Meters | | | | |
| 9 | 08/17/1965 16:46 | cm | | | | |
| 10 | 08/17/1965 16:47 | mm | | | | |
| | | PSI | | | | |
| | | 2,322.20 | | | | |

OUTPUT page—Site Information

Construction, depths, material, and site information are specified on the OUTPUT page (Figure 4). Most of this information is descriptive and does not affect estimated transmissivities. Aquifer material defines broad ranges of permissible transmissivities, which users should expand or replace with site specific limits. Pumping start time, recovery start time, and discharge rate affect transmissivity estimates and are specified in cell F37:F39.

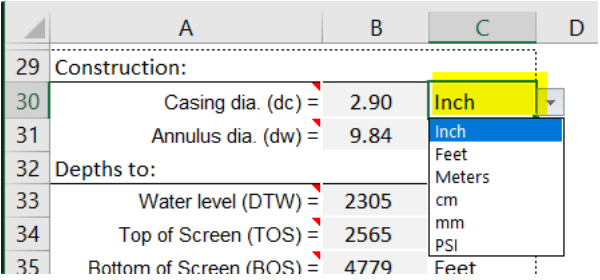
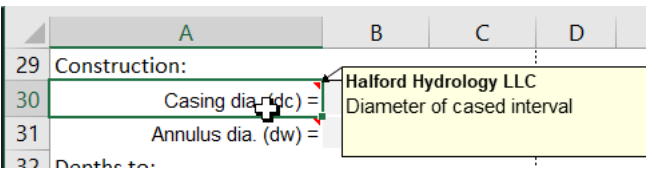
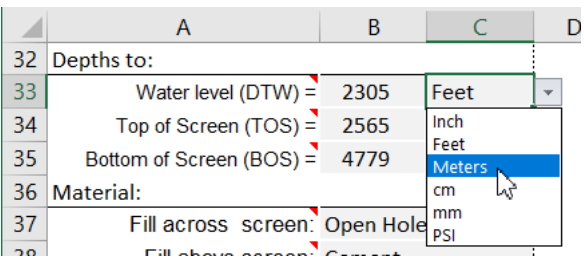
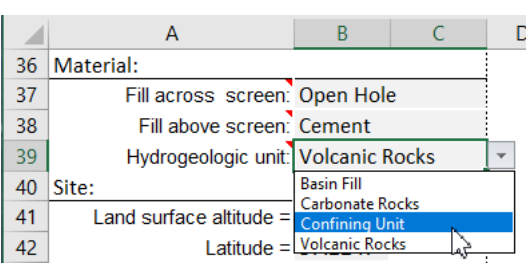
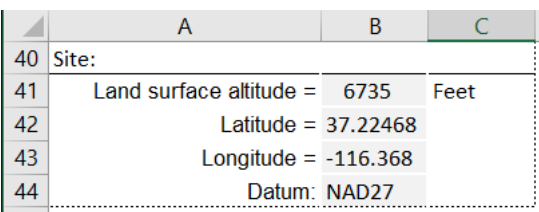
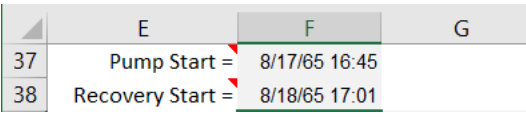
| | A | B | C | D | E | F | G | |
|----|--|----------------|------|---|-----------------------------|-----------------------------|---|--------------------|
| 25 | WELL ID: UE-19fS (2565-4779 ft) | | | | | GROSS FIT | | ADJUST LINE |
| 27 | Input is consistent | | | | | | | |
| 29 | Construction: | | | | | | | |
| 30 | Casing dia. (dc) = | 2.90 | Inch | | | | | |
| 31 | Annulus dia. (dw) = | 9.84 | Inch | | | | | |
| 32 | Depths to: | | | | | | | |
| 33 | Water level (DTW) = | 2305 | Feet | | | | | |
| 34 | Top of Screen (TOS) = | 2565 | Feet | | | | | |
| 35 | Bottom of Screen (BOS) = | 4779 | Feet | | | | | |
| 36 | Material: | | | | | | | |
| 37 | Fill across screen: | Open Hole | | | | | | |
| 38 | Fill above screen: | Cement | | | | | | |
| 39 | Hydrogeologic unit: | Volcanic Rocks | | | | | | |
| 40 | Site: | | | | | | | |
| 41 | Land surface altitude = | 6735 | Feet | | | | | |
| 42 | Latitude = | 37.2247 | | | | | | |
| 43 | Longitude = | -116.37 | | | | | | |
| 44 | Datum: | NAD27 | | | | | | |
| | | | | | Transmissivity = | 1160 feet ² /day | | |
| | | | | | Average K = | 1.16 ft/d | | |
| | | | | | Assumed thickness = | 1000 Feet | | |
| | | | | | Buffer: | 0:01 hr:min | | |
| | | | | | Pump Start = | 8/17/65 16:45 | | |
| | | | | | Recovery Start = | 8/18/65 17:01 | | |
| | | | | | Q-constant = | 130 GPM | | |
| | | | | | Significant Digits = | 2 | | |

Figure 4.—Site information for single-well aquifer test in the CJ-Drawdown+Recovery-2019.xlsm workbook.

Site Information

Enter site identifier in cell B25.

| | A | B | C | D |
|----|--|---|---|---|
| 25 | WELL ID: UE-19fS (2565-4779 ft) | | | |

| | |
|--|--|
| <p>Diameters of casing and annulus are specified in cells B30 and B31. Select units from pull-down menu in cell C30.</p> |  |
| <p>Commented cells further describe expected input.</p> |  |
| <p>Depths to static water level, top of screen, and bottom of screen are specified in cells B33, B34, and B35. Select units from pull-down menu in cell C33.</p> |  |
| <p>Filled annular material across screen and above screen are specified in cells C37 and C38.</p> <p>Hydrogeologic unit is specified in cell C39 and defines range of permissible transmissivities.</p> <p>Select materials from pull-down menus in cells C37:C39.</p> |  |
| <p>Site information is for completeness of reporting and is not otherwise used.</p> |  |
| <p>Starting times for pumping and recovery are specified in cells F37 and F38. These times define the pumping period.</p> <p>Recovery-start time divides data into drawdowns during pumping and residual drawdowns during recovery.</p> |  <p>Shows as</p> |

| | |
|---|--|
| | |
| <p>Constant flow rate is specified in cell F39 and units are selected from pull-down menu in cell G39.</p> <p>Significant digits specified in cell F41 affects reported transmissivity (T) and average hydraulic conductivity (K) in cells F30 and F31.</p> | |
| <p>Error conditions are reported in row 27. "Input is consistent," is reported when no errors exist.</p> | |

OUTPUT page—Estimating transmissivity

Drawdown and residual drawdowns are plotted and analyzed on the OUTPUT page (Figure 5). Pumping and depths to water are shown on a Cartesian plot, where drawdown and recovery data are differentiated. Early drawdowns and residual drawdowns can be excluded so that wellbore storage effects are not interpreted. A straight line initially is regressed to drawdown and recovery data with the “GROSS FIT” button. Fit between straight line and plotted data can be refined visually to ignore outliers with the “ADJUST LINE” button. Transmissivity is reported with a user defined number of significant digits. Average hydraulic conductivity is reported if an assumed thickness is specified.

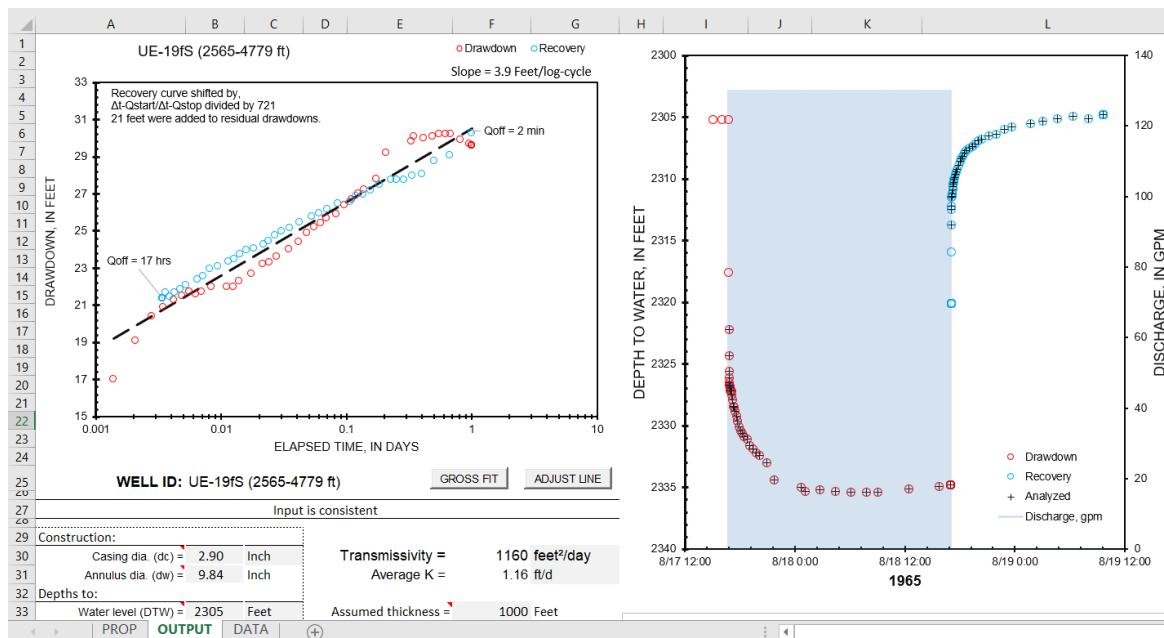
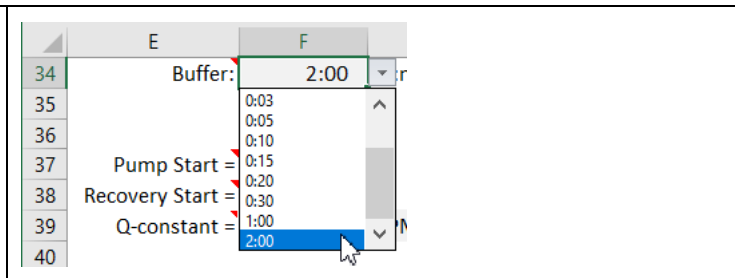


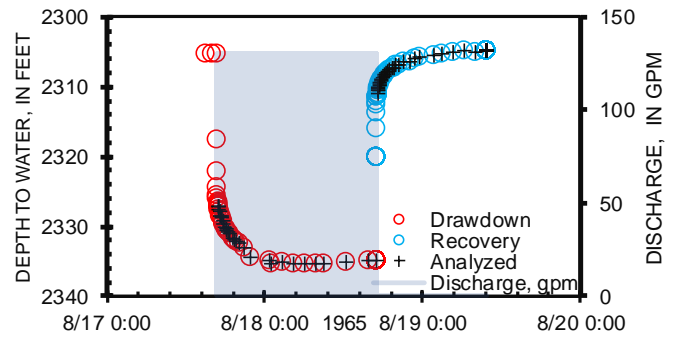
Figure 5.—Estimating transmissivity from single-well aquifer test in the CJ-Drawdown+Recovery-2019.xlsm workbook.

Estimating Transmissivity

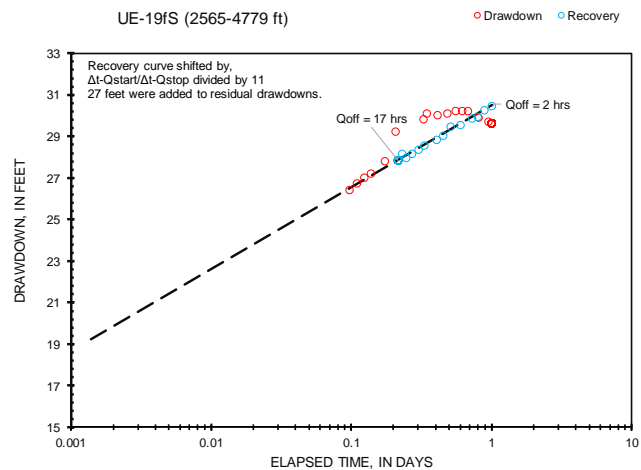
Buffer period in cell F34 changes analyzable data, where drawdowns and residual drawdowns in the buffer periods at starts of pumping and recovery are excluded from analysis.



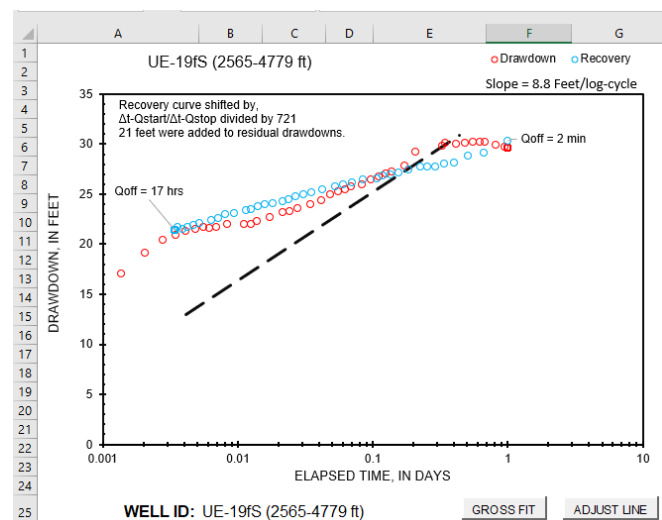
Buffered drawdowns and residual drawdowns that are not analyzed appear as open circles on the Cartesian time series.



Analyzed drawdowns and residual drawdowns appear on the semi-log plot. Analyzed data in Cartesian and semi-log plots change dynamically as buffer period is changed in cell F34.

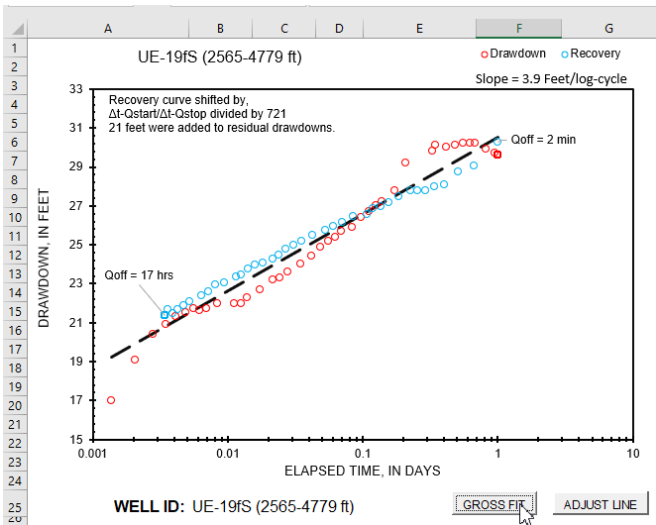


Prior to pressing “GROSS FIT” button (cell F25).



After pressing “GROSS FIT” button (cell F25) regresses a straight line to drawdown and recovery data.

Minimum and maximum values of Y-axis are redefined to bracket drawdown data, recovery data, and fitting line.

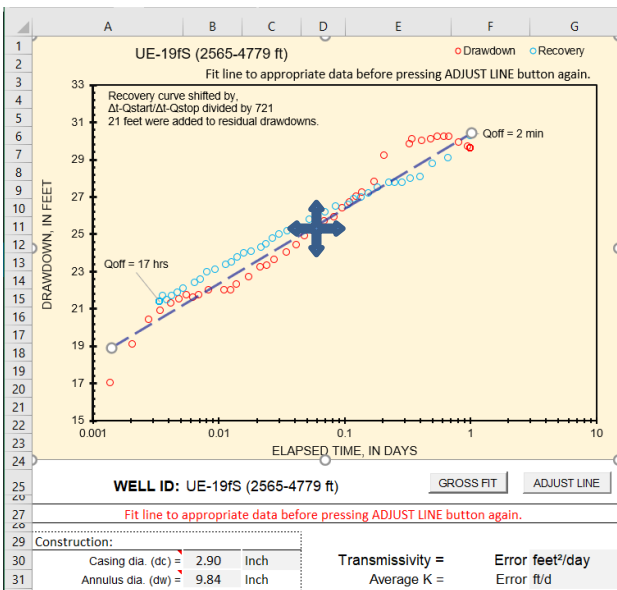


Pressing the “ADJUST LINE” button (cell G25).

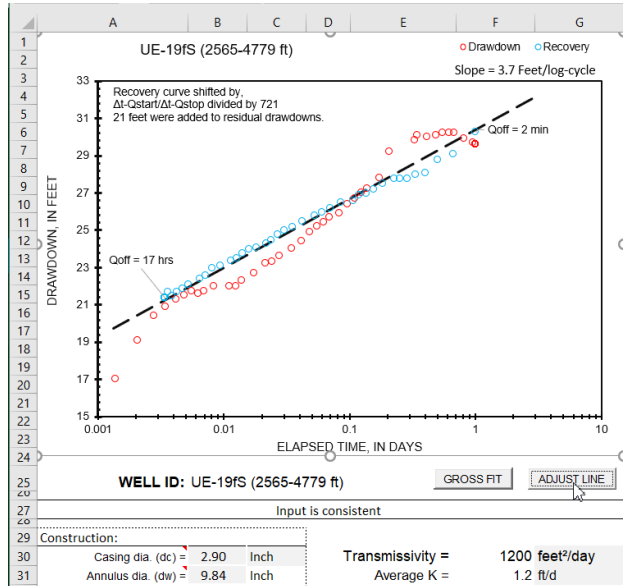
Converts the regression line in the chart to a graphical line that can be moved and adjusted.

Background color changes and remains changed while in fitting mode.

An error condition exists while graphically fitting line.



Pressing “ADJUST LINE” button (cell G25) again. Reverts graphical line to regression line and restores background color of chart.



Average K is not reported if an assumed thickness is not specified in cell F33.

| D | E | F | G |
|----|---------------------|------|-----------|
| 30 | Transmissivity = | 1200 | feet²/day |
| 31 | | | |
| 32 | | | |
| 33 | Assumed thickness = | | Feet |

Units of reported transmissivity are selected from pull-down menu in cell G30.

| D | E | F | G |
|----|------------------|-----|------------|
| 30 | Transmissivity = | 120 | meter²/day |
| 31 | Average K = | 1.2 | feet²/day |
| 32 | | | meter/day |
| | | | gpd/ft |

Units of reported hydraulic conductivity (K) are selected from pull-down menu in cell G31.

| D | E | F | G |
|----|---------------------|-------|-----------|
| 30 | Transmissivity = | 1200 | feet²/day |
| 31 | Average K = | 4E-13 | m² |
| 32 | | | ft/d |
| 33 | Assumed thickness = | 1000 | m/d |
| 34 | Buffer: | 0:01 | cm/d |
| 35 | | | cm/s |
| 36 | | | m/s |
| | | | m |

PROP page

Annular fill, grouts, and hydrogeologic units are specified on the PROP page (Figure 6). Annular fill and grout are descriptive lists in columns A and B that can be edited by the user. Hydrogeologic units define a range of permissible transmissivities for each hydrogeologic unit. The default list was defined for southern Nevada in the Death Valley system from 269 aquifer tests and specific-capacity estimates (Figure 7). The list of hydrogeologic units should be adapted to specific information from a user's study area.

| | A | B | C | D | E | F | G |
|---|--------------|-----------|---|--------------------|--------------------------|--------------------------|---|
| 1 | Annular Fill | GROUTS | | Hydrogeologic Unit | Tmin, ft ² /d | Tmax, ft ² /d | |
| 2 | Gravel | Bentonite | | Basin Fill | 5 | 60,000 | |
| 3 | Coarse Sand | Cement | | Carbonate Rocks | 2 | 400,000 | |
| 4 | Medium Sand | Backfill | | Confining Unit | 0.00002 | 1,900 | |
| 5 | Fine Sand | Open Hole | | Volcanic Rocks | 0.0001 | 200,000 | |
| 6 | Open Hole | | | | | | |
| 7 | | | | | | | |

Figure 6.—Annular fills, grouts, and ranges of transmissivities on the PROP page in the CJ-Drawdown+Recovery-2019.xlsm workbook.

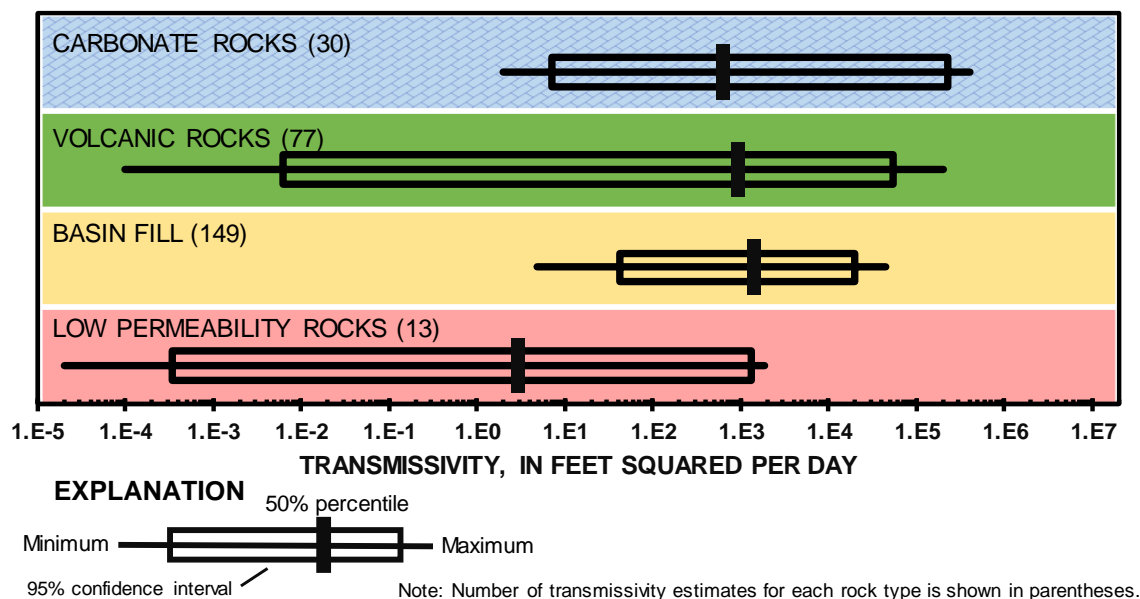


Figure 7.— Minimum, maximum, median, and 95-percent confidence interval of log-transmissivities in four rock-type categories from 269 field estimates of transmissivity in the Death Valley regional flow system (Halford and Jackson, 2019).

Halford, K.J., and T.R. Jackson, 2019, Groundwater Characterization and Effects of Pumping in the Death Valley Regional Groundwater Flow System, Nevada and California, with Special Reference to Devils Hole: U.S. Geological Survey Professional Paper 2019–XXXX, xx p.