

## SeriesSEE verification folder

Analytical solutions that were computed with the FORTRAN program WLmodel and published results of the same solutions are stored by subfolder in the SeriesSEE verification folder. The analytical models for pneumatic lag, gamma, moving average, step, Theis, and tide are verified against known solutions in the subfolders AirLAG, Gamma, MovingAverage, Step, Theis, and Tide, respectively (Figure 1). Copies of published programs are in their respective subfolders, so all batch files should function without modification on a Windows platform.

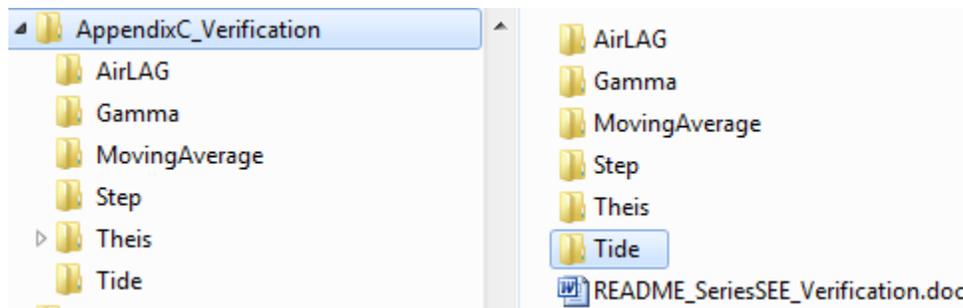


Figure 1.—Root directory and subfolders for verification of analytical solutions in SeriesSEE.

### ***AirLAG***

The analytical solution for propagation of barometric changes through the unsaturated zone is equivalent to surface-water changes affecting groundwater levels in a saturated, confined aquifer. Output from the FORTRAN program WLmodel that is called by SeriesSEE was compared to the solution of Barlow and Moench (1998) for a one-dimensional confined aquifer. The Barlow and Moench (1998) solution is in the program [stlk1.exe](#) (Figure 2).

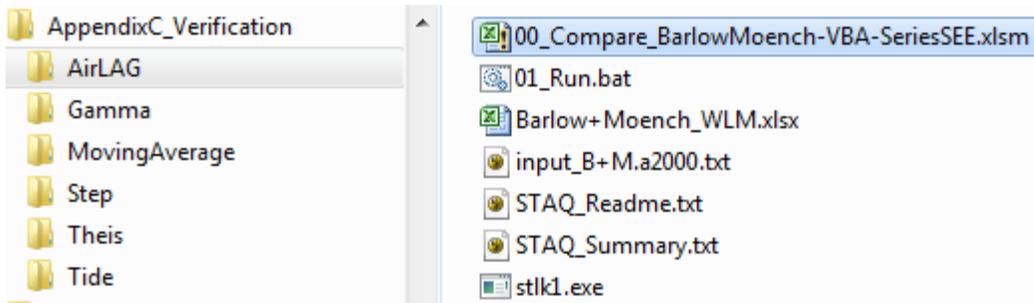


Figure 2.—Contents of the .AirLAG subfolder.

Water-level changes 2,000 ft from a variable-head boundary were solved with Barlow and Moench (1998), a VBA solution, and WLmodel (SeriesSEE). All three solutions agreed within 0.003 ft, which was relative to a maximum simulated change of 0.86 ft (Figure 3). The comparison was limited to 3 days where stage at  $x = 0$  ft was displaced 1 ft between 0.3 and 1.0 days (Figure 3). The SeriesSEE archive is in the file Barlow+Moench\_WLM.xlsx.

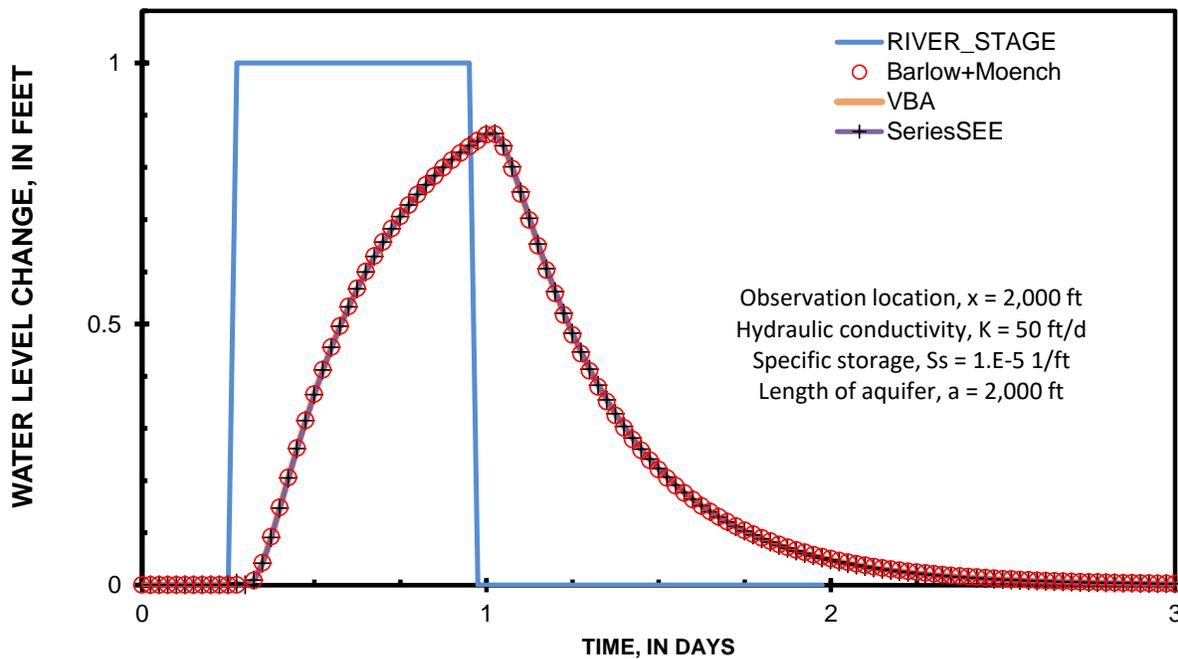


Figure 3.—Comparison between Barlow and Moench (1998), VBA, and WLmodel (SeriesSEE) solutions.

## Gamma

The transfer function of the Water-Balance/Transfer Function (WBTF) model was selected because delayed and attenuated water-level rises can be computed from infiltration (O'Reilly, 2004). The gamma transform retains the transfer function from the WBTF model that translates a discrete pulse of infiltration below the root zone to recharge at the water table. The delay between infiltration and recharge at the water table increases as unsaturated zone thickness increases. Recharge pulses also are attenuated and prolonged as unsaturated zone thickness increases.

Output from the FORTRAN program WLmodel that is called by SeriesSEE was compared to the solution of O'Reilly (2004) for transferring infiltration to recharge. The O'Reilly (2004) solution is in the program wbt.exe, <http://pubs.usgs.gov/sir/2004/5195/> (Figure 4).

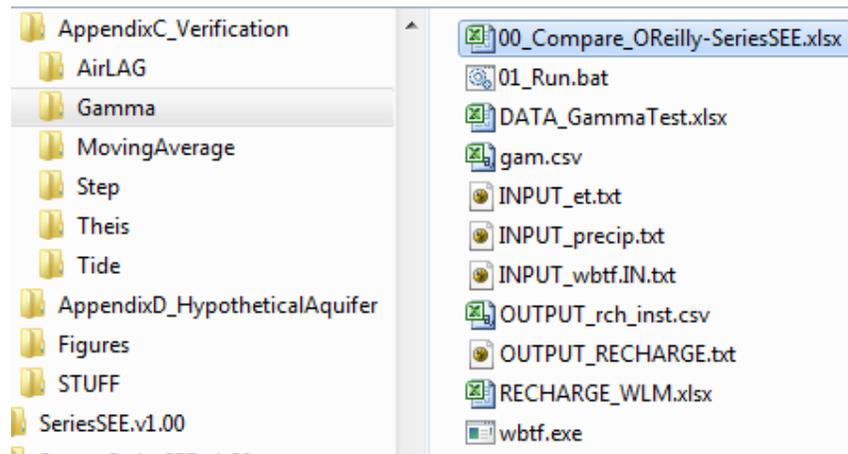


Figure 4.—Contents of the .Gamma subfolder.

Recharge rates from a series of infiltration events to an unsaturated zone, which was characterized by  $n = 1$  and  $k = 5$  d were solved with O'Reilly (2004) and WLmodel (SeriesSEE). The maximum difference between solutions was 0.29 mm/d, while simulated recharge rates were 19 mm/d (Figure 5). Differences exist because O'Reilly (2004) uses average recharge rates during infiltration events, truncates the number of events summed, and preserves the area of the truncated, gamma function. The SeriesSEE archive is in the file RECHARGE\_WLM.xlsx.

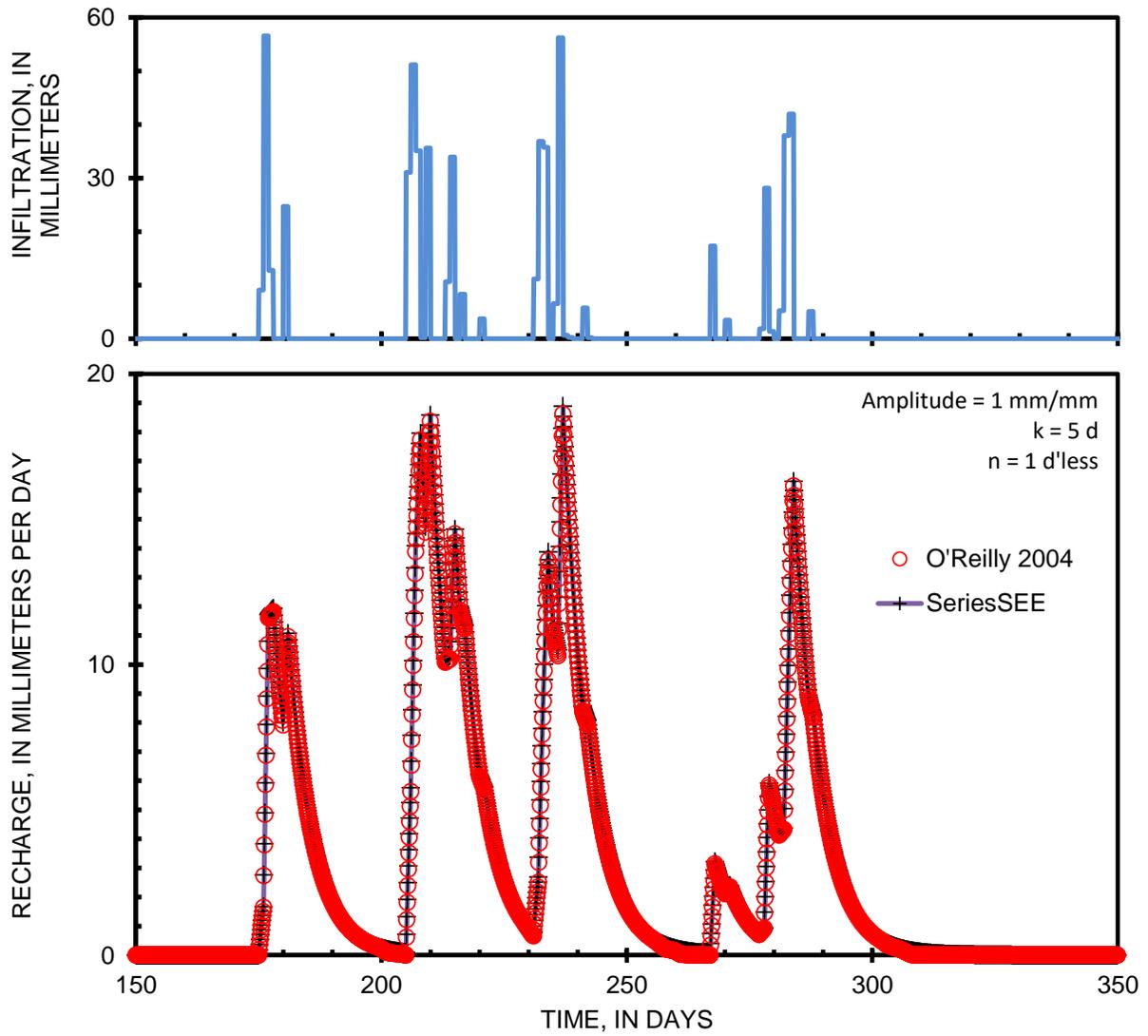


Figure 5.—Comparison between O’Reilly (2004) and WLmodel (SeriesSEE) solutions.

## MovingAverage

Moving averages from the FORTRAN program WLmodel, which is called by SeriesSEE, were compared to moving averages computed with the AVERAGE function in Microsoft® Excel. Moving average comparisons were created in the workbook 00\_Compare\_MovingAVG\_Excel-SeriesSEE.xlsm (Figure 6). Both solutions agreed within machine precision of less than  $10^{-7}$  ft where water levels fluctuated more than 0.1 ft (Figure 7). The SeriesSEE archive is in the file Excel\_12-hr\_WLM.xlsx.

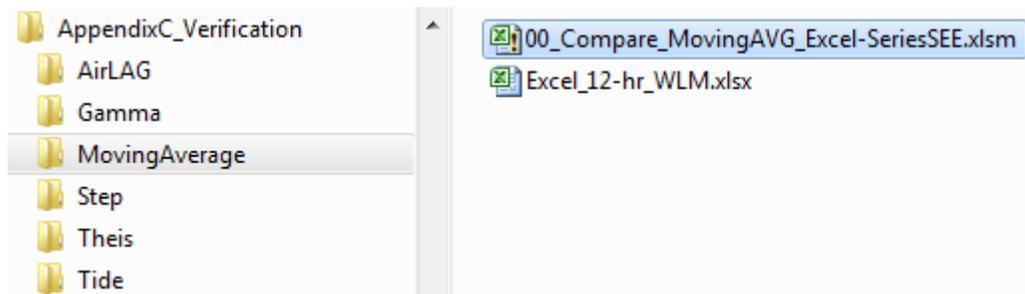


Figure 6.—Contents of the .MovingAverage subfolder.

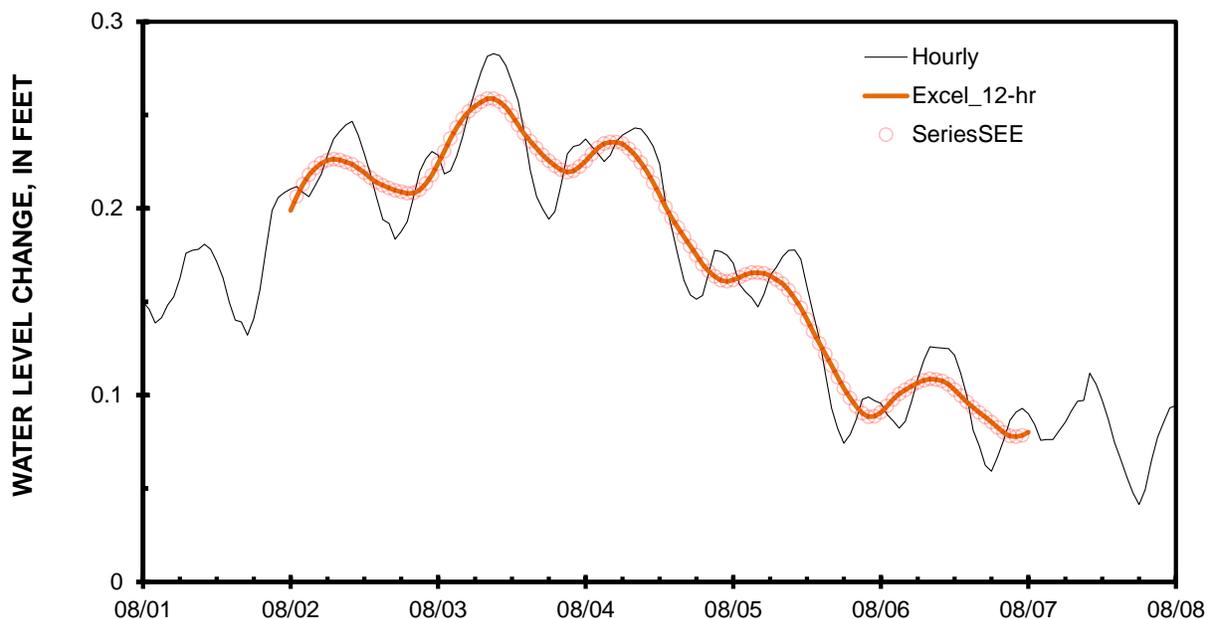


Figure 7.—Original series component of hourly measurements and comparison between intrinsic AVERAGE function in Microsoft® Excel and WLmodel (SeriesSEE) solution.

## Step

Step changes from the FORTRAN program WLmodel, which is called by SeriesSEE, were compared to step changes computed with an IF function in Microsoft® Excel. Step change was compared in the workbook 00\_Compare\_Step\_Excel-SeriesSEE.xlsm (Figure 8). Both solutions agreed within machine precision of less than  $10^{-7}$  ft where the step change was +0.17 ft (Figure 9). The SeriesSEE archive is in the file Excel\_Step\_WLM.xlsx.

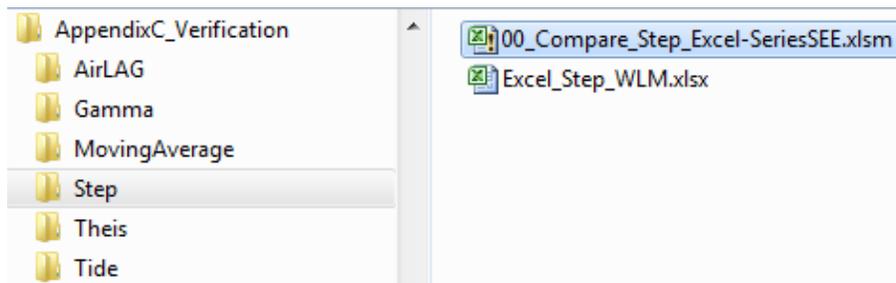


Figure 8.—Contents of the .\Step subfolder.

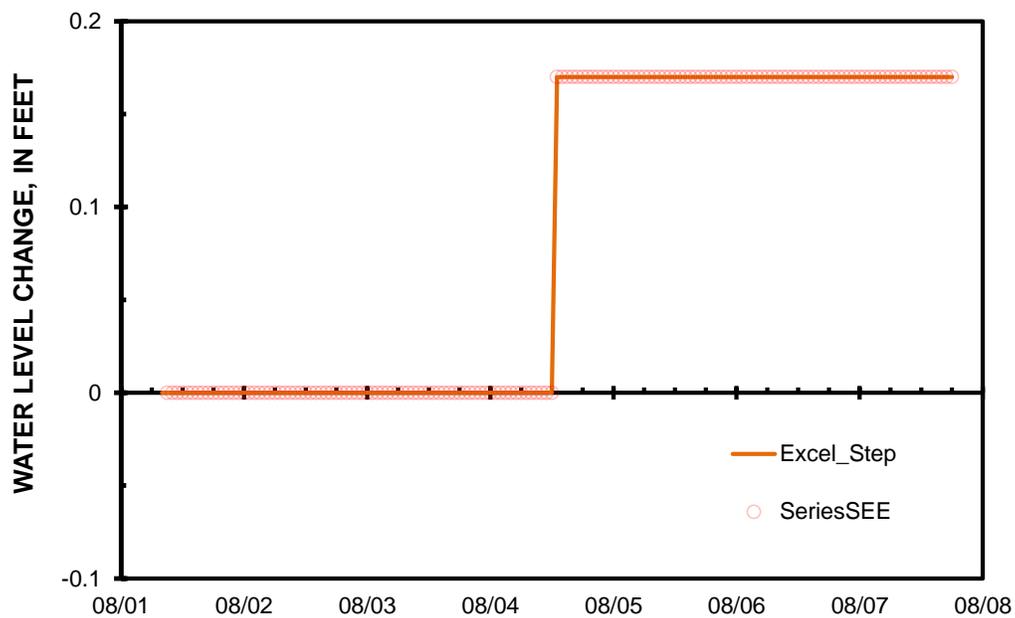
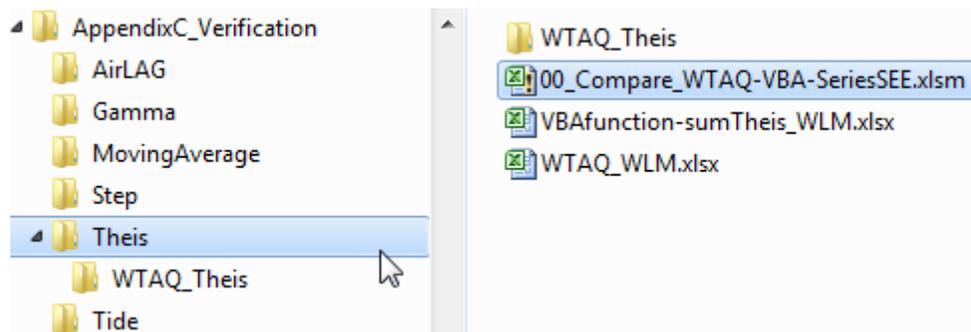


Figure 9.—Comparison between intrinsic IF function in Microsoft® Excel and WLmodel (SeriesSEE) solution of step change.

## ***Theis***

Theis-transform results from the FORTRAN program WLmodel that is called by SeriesSEE were compared to equivalent solutions of Wenzel (1942), WTAQ (Barlow and Moench, 1999), and VBA functions (Hunt, 2005) for a one-dimensional, radial flow in a confined aquifer. Wenzel (1942) is the first tabulation of well-function responses for dimensionless time,  $u$ . WTAQ (Barlow and Moench, 1999) solves for two-dimensional unconfined flow, but also solves the simpler Theis solution. The WTAQ program, wtaq\_1.2.exe, input, and output files are under the WTAG\_Theis subfolder, (Figure 10). A VBA solution of the Theis well function, wtheis, and other supporting VBA functions are in the file 00\_Compare\_WTAQ-VBA-SeriesSEE.xlsm (Figure 10).



**Figure 10.—Contents of the .Theis subfolder.**

The well function, **W\_1**, is a VBA function that is the basis of VBA functions and FORTRAN functions that apply Theis-transforms to pumping schedules. The algorithm in the well function was developed by [Hunt \(2005\)](#). The algorithm was verified against results from Wenzel (1942, p. 88-89) and a brute-force solution of the exponential integral in the VBA function **wTheis**. Results from the solutions agree within the tolerances of the tabulated values (Figure 11).

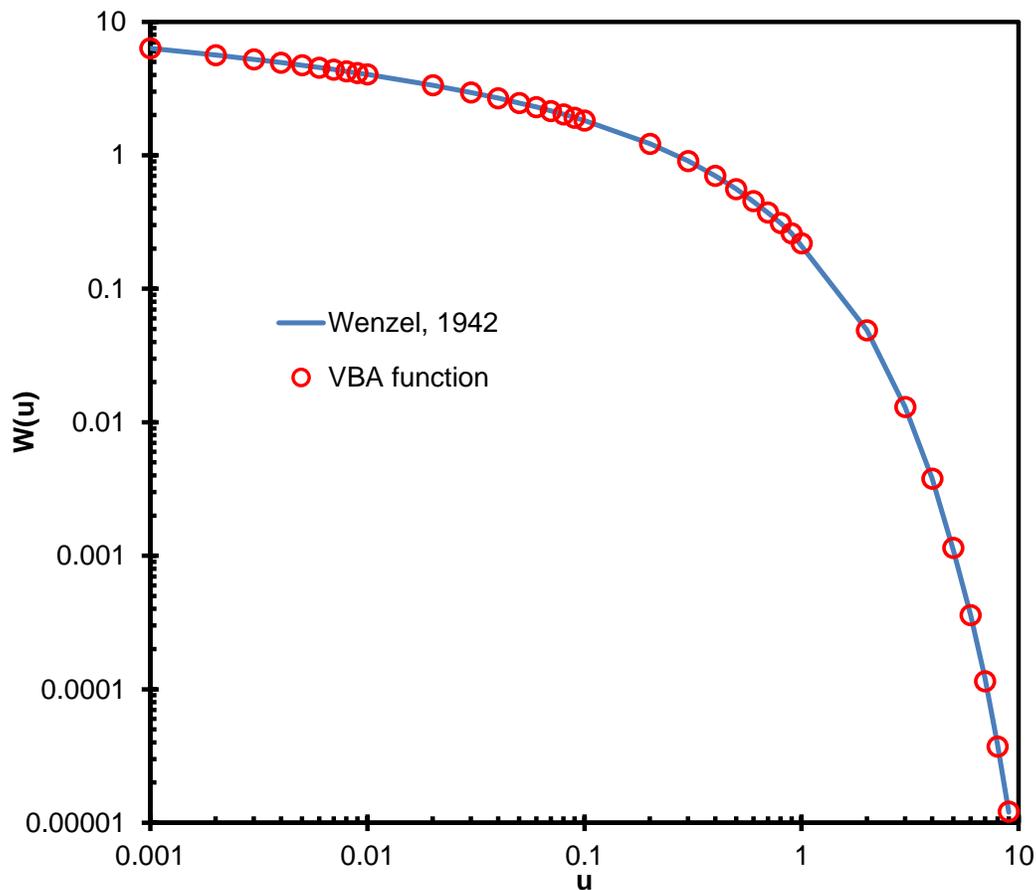


Figure 11.—Comparison between Wenzel (1942) and VBA function  $W_1$  that solve the well function,  $W(u)$  for the Theis equation as a function of dimensionless time,  $u$ .

Drawdown in an observation well 500 ft away from a well pumping 300 gpm was calculated with Wenzel (1942), the VBA function *sTheis*, WTAQ (Barlow and Moench, 1999), and WLmodel from SeriesSEE (Figure 12). The VBA function *sTheis* computes drawdown with a flow rate, transmissivity, storage coefficient, radial distance, and elapsed time. The standard deviation between *sTheis*, WTAQ, and WLmodel solutions averaged 0.1 percent of the average, simulated drawdowns. The SeriesSEE archive is in the file WTAQ\_WLM.xlsx.

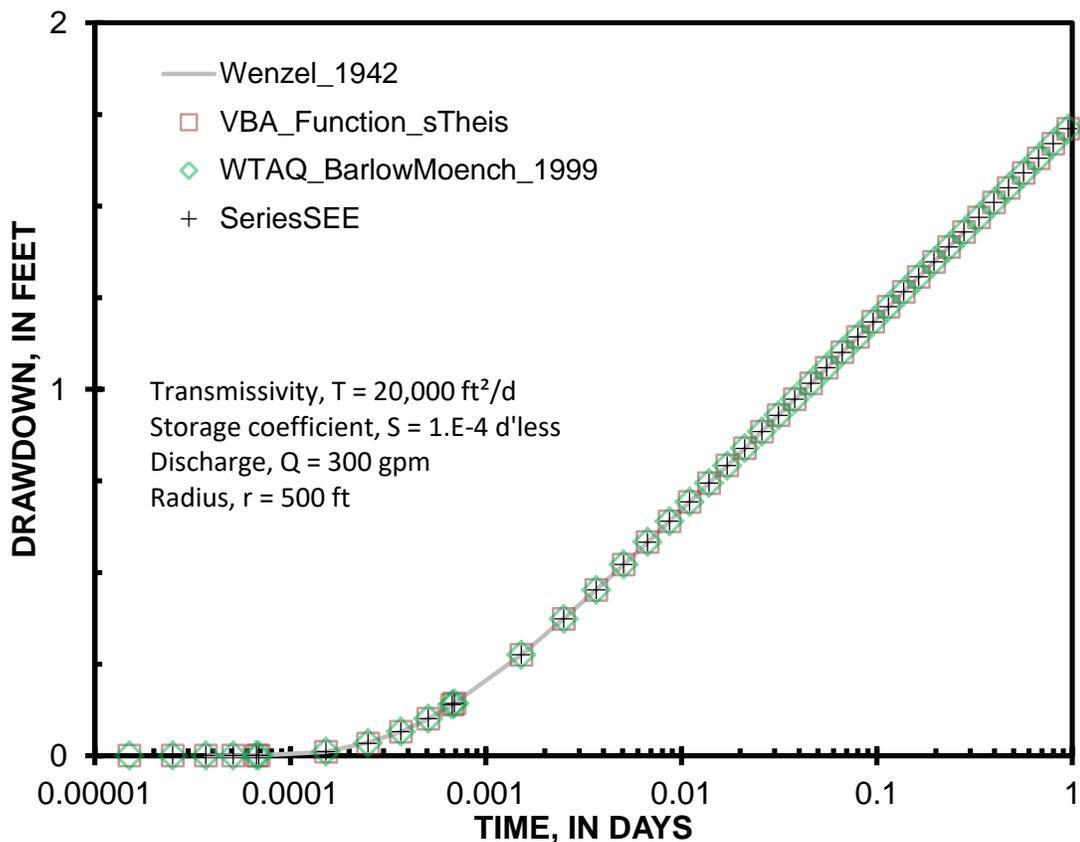


Figure 12.—Comparison between Wenzel (1942), VBA function *sTheis*, WTAQ (Barlow and Moench, 1999), and WLmodel (SeriesSEE) solutions of 1 day of pumping.

Drawdown in an observation well 2,000 ft away from a well with a variable pumping schedule was calculated with the VBA function **SUMTheis**, and WLmodel from SeriesSEE (Figure 13). The VBA function **SUMTheis** computes drawdown from a pumping schedule given the date and time of evaluation. A pumping schedule is converted to a series of changes in pumping rates and elapsed times from each change. Drawdown from each incremental change is computed with the **sTheis** function and summed in the **SUMTheis** function. The **SUMTheis** functions are similar in the VBA and FORTRAN versions. The maximum absolute difference between the VBA and FORTRAN functions was 0.00002 ft. The SeriesSEE archive is in the file VBAfunction-sumTheis\_WLM.xlsx.

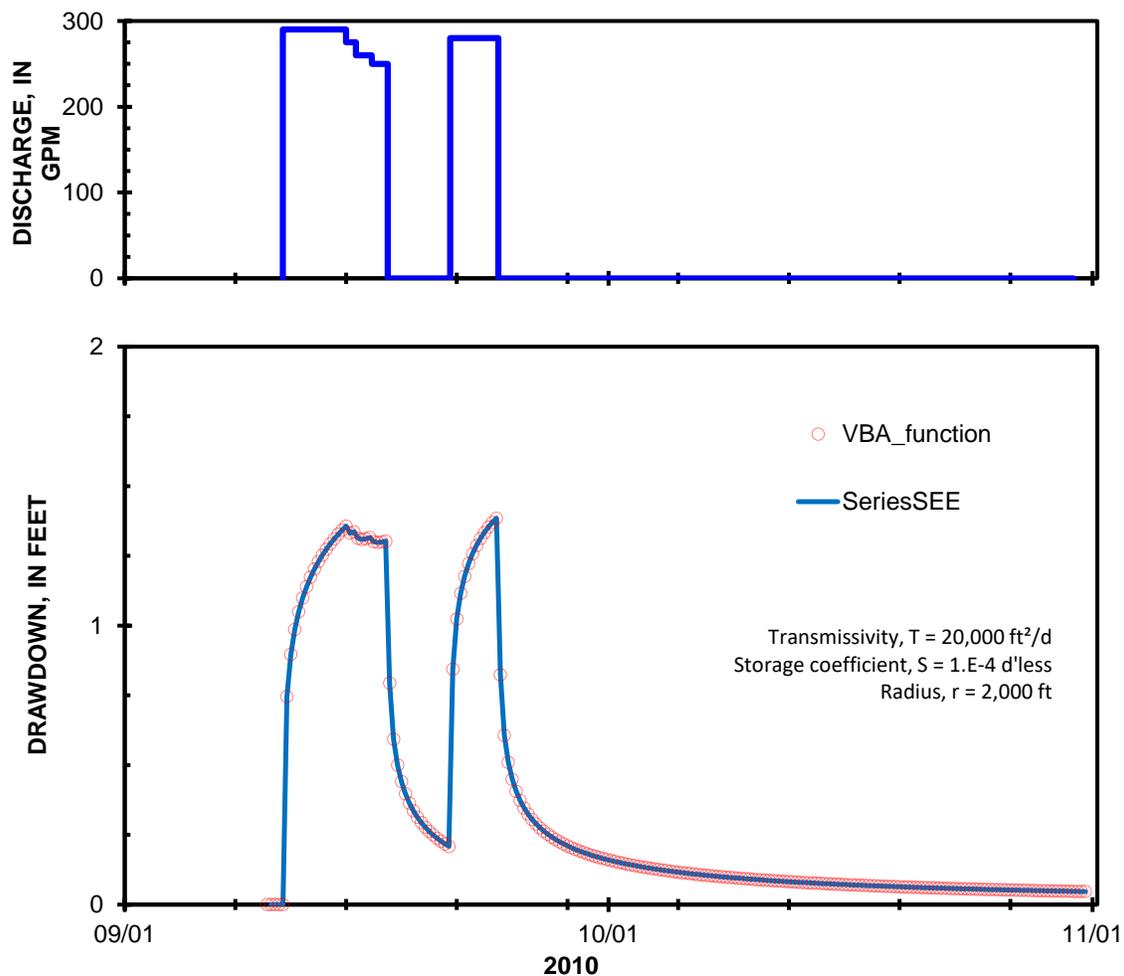


Figure 13.—Comparison between VBA function *sumTheis* and WLmodel (SeriesSEE) solutions for variable pumping rates between 0 and 290 gpm.

## Tide

Gravity, tilt, and dry tides from the program WLmodel were compared to *etide* (Harrison, 1971) and VBA functions (Halford, 2006). WLmodel is a FORTRAN program that is called by SeriesSEE. The program *etide*, *etide.exe*, input, and output files are under the Tide subfolder (Figure 14). Input to *etide* is created by the batch file 01\_Run.bat prior to executing the program *etide*. VBA solutions of the gravity, tilt, and dry tides are solved by the functions; *dGravityTide*, *dTiltTide*, and *dDryTide*, respectively, in the file 00\_Compare\_Etide-VBA-SeriesSEE.xlsm (Figure 14). Source code for the FORTRAN program *etide* is in the subfolder Etide\_FORTRAN\_Harrison-1971.

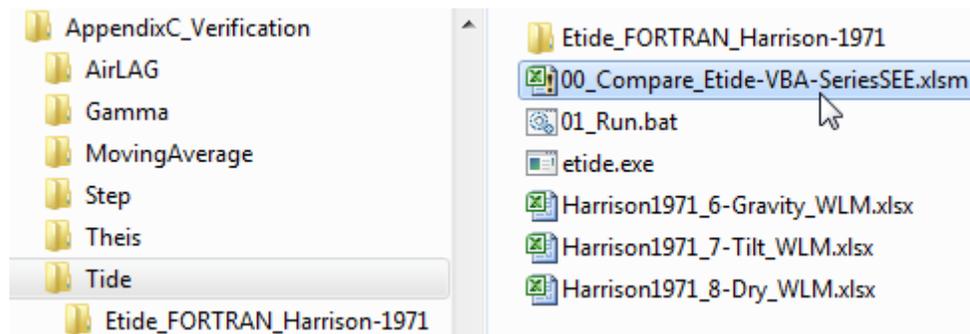


Figure 14.—Contents of the .Tide subfolder.

Gravity, tilt, and dry tides at a latitude of  $36^\circ$ , longitude of  $-117^\circ$ , and altitude of 0 m were calculated with *etide* (Harrison, 1971), VBA function, and WLmodel from SeriesSEE (Figure 15). Tides from *etide* (Harrison, 1971), VBA function, and WLmodel agreed within 0.0001 percent of range of simulated tides. Differences exist because Julian dates were truncated after six decimal places in the output from the program *etide*. The SeriesSEE archives for gravity, tilt, and dry tides are in the files Harrison1971\_6-Gravity\_WLM.xlsx, Harrison1971\_7-Tilt\_WLM.xlsx, and Harrison1971\_8-Dry\_WLM.xlsx, respectively.

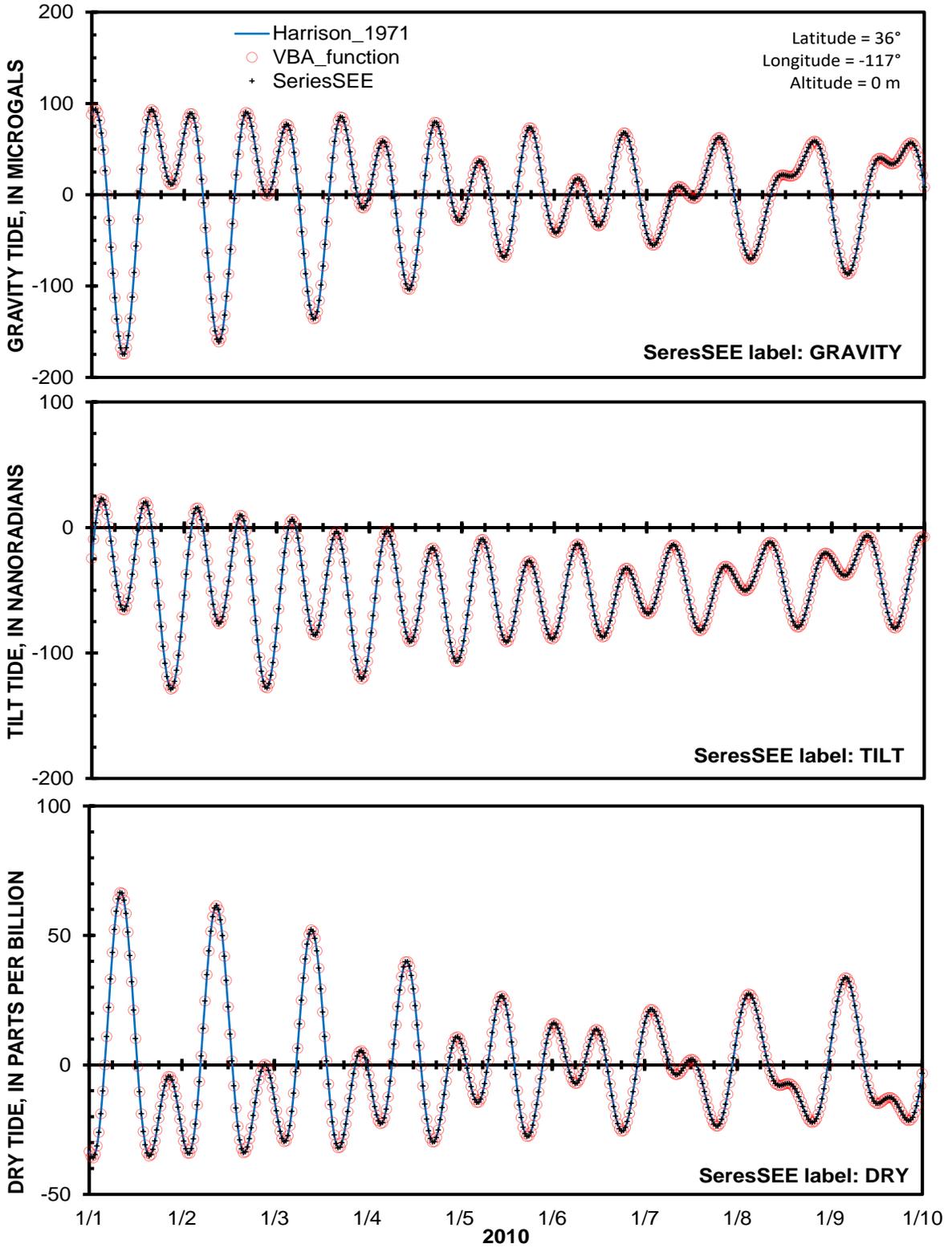


Figure 15.—Comparison between etide (Harrison, 1971), VBA functions, and WLmodel (SeriesSEE) solutions of gravity, tilt, and dry tides.

## References

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