ground water

Technical Note/

Interpretation of Transmissivity Estimates from Single-Well Pumping Aquifer Tests

by Keith J. Halford[1], Willis D. Weight[2], and Robert P. Schreiber[3]

Abstract

Interpretation of single-well tests with the Cooper-Jacob method remains more reasonable than most alternatives. Drawdowns from 628 simulated single-well tests where transmissivity was specified were interpreted with the Cooper-Jacob straight-line method to estimate transmissivity. Error and bias as a function of vertical anisotropy, partial penetration, specific yield, and interpretive technique were investigated for transmissivities that ranged from 10 to 10,000 m^2/d. Cooper-Jacob transmissivity estimates in confined aquifers were affected minimally by partial penetration, vertical anisotropy, or analyst. Cooper-Jacob transmissivity estimates of simulated unconfined aquifers averaged twice the known values. Transmissivity estimates of unconfined aquifers were not improved by interpreting results with an unconfined aquifer solution. Judicious interpretation of late-time data consistently improved estimates where transmissivity exceeded 250 m^2/d in unconfined aquifers.

Introduction

Single-well aquifer tests provide estimates of transmissivity where cost and access preclude multiwell aquifer tests. This is particularly true where depth to water is significant as frequently occurs in the arid West (Belcher et al. 2001). Hydraulic properties other than transmissivity can only be quantified using multiple observation wells or flow logs (Hanson and Nishikawa 1996).

Single-well aquifer tests frequently are analyzed with the Cooper-Jacob (1946) method because of its simplicity. Transmissivity is estimated by fitting a straight line to drawdowns on an arithmetic axis vs. time on a logarithmic axis in a semilog plot. Drawdowns in confined and unconfined aquifers have been analyzed by many practitioners using the Cooper-Jacob method, regardless of differences between field conditions and theory.

As the Cooper-Jacob method is a simplification of the Theis solution, the pumping well should fully penetrate a confined, homogeneous, and isotropic aquifer. Single-well tests from partially penetrating wells in unconfined aquifers depart greatly from the Theis (1935) model. Moreover, unconfined aquifer tests are affected by vertical anisotropy and specific yield in addition to transmissivity and storage coefficient. These additional parameters control vertical gradients that are created by partial penetration and drainage from the water table. Likewise, leakage from adjacent confining beds also could affect transmissivity estimates, which likely will be overestimated by the Cooper-Jacob method.

Transmissivity estimates from single-well tests in unconfined aquifers also are affected by discharge rate, test duration, and interpretive technique. The transition from the release of water from storage owing to the compressibility of the medium and fluid to drainage of pores is less likely to be observed during a test of relatively small discharge or short duration. Interpretation of drawdown data is hampered because a final drawdown limb, which has a slope predicted by the Cooper-Jacob method, is absent.

Purpose and Scope

This article documents how interpreting single-well pumping aquifer tests with the Cooper-Jacob method affects transmissivity estimates. Drawdowns in 800

[1]Corresponding author: USGS, 333 West Nye Lane, Room 203, Carson City, NV 89706; (775) 887-7613; khalford@usgs.gov

[2]Montana Tech of The University of Montana, 1300 West Park Street, Butte, MT 59701; (406) 496-4329; wweight@mtech.edu

[3]CDM, One Cambridge Place, 50 Hampshire Street, Cambridge, MA 02139; (617) 452-6251; schreiberrp@cdm.com

Received January 2005, accepted August 2005.

Journal compilation © 2006 National Ground Water Association.
No claim to original US government works.

doi: 10.1111/j.1745-6584.2005.00151.x

single-well tests were simulated with radial MODFLOW models using transmissivities between 10 and 10,000 m^2/d. Drawdowns from 628 of these simulated tests maintain water levels above pump intakes and were interpreted with the Cooper-Jacob straight-line method to calculate transmissivity. Transmissivity error and bias as a function of vertical anisotropy, partial penetration, specific yield, and interpretive technique were investigated.

Approach

Effects of unmet assumptions on the reliability of Cooper-Jacob transmissivity estimates were investigated by interpreting 628 simulated drawdowns where transmissivity was specified. The transmissivity of each homogeneous aquifer with an impermeable lower boundary was estimated with a Cooper-Jacob analysis of simulated drawdowns. Effects of unmet assumptions were quantified by comparing Cooper-Jacob transmissivity estimates to values specified in the simulations. These specified transmissivities are herein referred to as known transmissivities, which were limited to plausible ranges. Transmissivities ranged from 10 to 10,000 m^2/d. Transmissivities <10 m^2/d were excluded because slug tests are more practical than pumping tests for aquifers of lower permeability. A specific storage of $5 \times 10^{-6}\ m^{-1}$ was assigned to all aquifers. Preliminary experiments showed that transmissivity estimates were insensitive to plausible specific storages. Specific yields ranged from 0.05 to 0.3. Vertical anisotropies ranged between 0.02 and 0.2 and were assumed to represent a sedimentary system.

Pumping wells penetrated between 10% and 100% of the aquifer thickness. All partially penetrating wells were open at the top of the aquifer or water table. Wellbore storage and skin effects of the pumping wells, which decreased maximum potential pumping rates, were also simulated. Simulation results were considered physically impossible and rejected where simulated water levels in the pumping well were <3 m above the bottom of the well. Wells in unconfined aquifers were simulated to penetrate 25% or more of the saturated thickness so that water levels would remain above the pump intakes.

All single-well tests were simulated over 2-d periods to balance testing effectiveness and operational constraints. Transmissivity estimates from tests of longer duration are less ambiguous because drainage from the water table will be observed and drawdowns will follow a late-time response. Actual single-well tests typically range between 1 and 3 d in duration because of operational constraints.

Simulated Aquifer Tests

All single-well aquifer tests were simulated with an axisymmetric, radial geometry in a one-layer MODFLOW model (McDonald and Harbaugh 1988; Harbaugh and McDonald 1996). That is, the horizontal dimension is represented by columns and the vertical dimension is represented by rows, so that the column, in effect, is laid on its side, or alternatively the one MODFLOW layer is conceptually flipped to the vertical position (Anderson and Woessner 1992, 175–176). The production well in each model was simulated as a high-conductivity zone where water was removed from the uppermost cell and flow was apportioned within MODFLOW (Halford 1997). All models extended 100,000 m from the production well along a row that was discretized into 99 columns (Figure 1). Pumping wells were located in column 1 and aquifer material was specified in columns 2 through 99. Column 2 was 0.02 m wide, and the remaining columns were 1.15 times the width of the previous column. The 100-m-thick aquifers were uniformly subdivided into 100 rows. All models had no-flow lateral boundaries, initial heads of 0 m, and a single stress period of 50 time steps. The first time step was 0.1 s in duration. Each successive time step was 1.3 times greater than the previous one.

Hydraulic properties were modified consistently to simulate axisymmetric radial flow. That is, simulated pumpage was withdrawn from column 1 so that distance from the left edge of column 1 to a column node would be equivalent to the radial distance from the pumping well. Hydraulic conductivity and storage of the ith column were multiplied by $2\pi r_i$ to simulate radial flow where r_i is the distance from the outer edge of the first column to the center of the ith column. Axisymmetric radial flow has been solved with MODFLOW by using many layers and a single row (Reilly and Harbaugh 1993; Clemo 2002). A single MODFLOW layer is more convenient because input is defined easily, all conductances are computed within the BCF package, and output is checked quickly.

Pumping rates were simulated near the maximum rate that could be pumped practically from an aquifer of a given transmissivity. Values of pumping rates in m^3/d were equal to transmissivity values in m^2/d. Realistic wellbore storage effects were simulated by increasing well diameters as pumping rates increased. A storage coefficient of 1 was assigned in row 1 of column 1 to simulate wellbore storage. Conductances between columns 1 and 2 were reduced 10-fold to simulate skin effect where a well was present.

Interpretation

Cooper-Jacob transmissivity was estimated for each single-well test by a mechanistic approach and separately

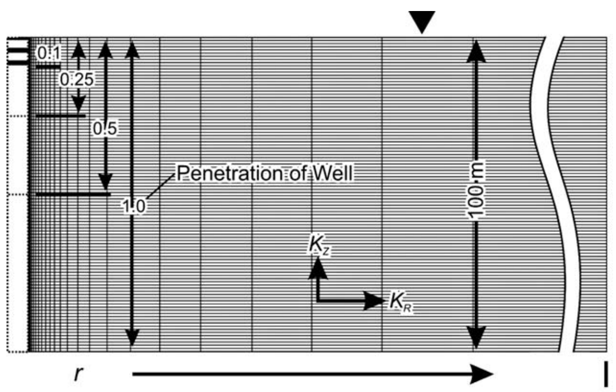

Figure 1. Definition sketch of hypothetical well and aquifer system with grid for numerical solution.

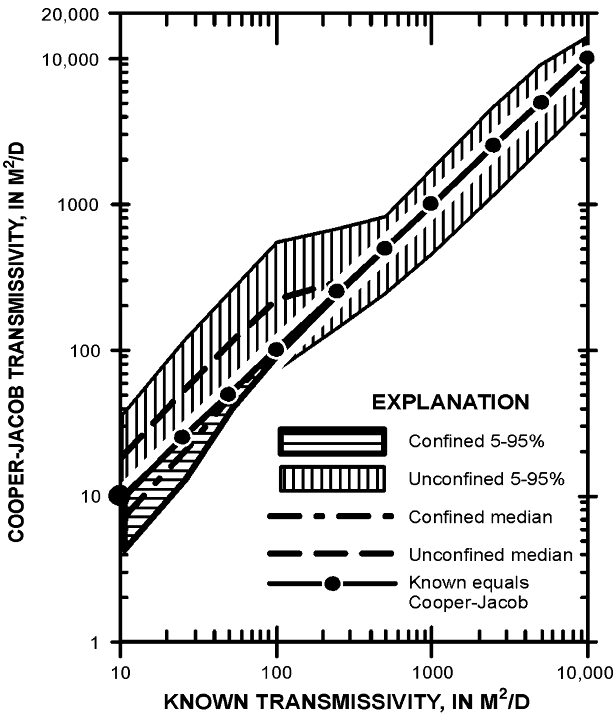

Figure 2. Comparison between known transmissivities and Cooper-Jacob estimates by an analyst for the 160 confined aquifers and 468 unconfined aquifers.

by six analysts, including the three authors and three volunteers. Transmissivity was estimated without interpretation by defining the semilog slope with drawdowns at 0.5 and 2 d after pumping started. A minimum time of 0.5 d was selected as a compromise between avoiding the early-time complications due to wellbore storage, partial penetration, and water table effects and assumed measurement sensitivity for actual aquifer tests. This noninterpretive approach is herein referred to as the mechanistic approach. Results from the mechanistic approach and novice analysts who incorrectly honor all data are similar. Experience guided individual analysts' best fit of semilog slopes to relevant data. All simulated drawdowns and analyses can be retrieved from http://nevada.usgs.gov/tech/groundwater_05.htm.

Confined Analysis

Cooper-Jacob transmissivity estimates in confined aquifers were affected minimally by partial penetration, vertical anisotropy, and interpretive technique (Figure 2). Known transmissivities of 50 m^2/d or greater were estimated within 20%. A steady additional drawdown from partial penetration and vertical anisotropy was established before 12 h of pumping had elapsed. This additional drawdown minimally affected all estimates of slope and transmissivity. Confined aquifer test results were unambiguous, and transmissivity estimates varied little among analysts.

Unconfined Analysis

Transmissivities in unconfined aquifers were overestimated with a mechanistic application of the Cooper-Jacob method. More than 75% of known transmissivity values between 10 and 1000 m^2/d were overestimated by a factor of 2. One percent of estimates were >10 times known transmissivity values. About 80% of known transmissivity values between 1000 and 10,000 m^2/d were estimated within a factor of 2.

More than 90% of transmissivities in unconfined aquifers were estimated within a factor of 2 by experienced analysts (Figure 2). Transmissivity estimates of relevant drawdowns by experienced analysts were more accurate than mechanistic estimates of transmissivity. Analysts provided better transmissivity estimates compared to the mechanistic approach for known transmissivity values that ranged between 250 and 5000 m^2/d. Analysts' interpretation did not significantly improve transmissivity estimates or remove bias for known transmissivity values that ranged between 10 and 100 m^2/d.

Interpretation was ambiguous in unconfined aquifers where transmissivity ranged between 10 and 250 m^2/d. For one aquifer, transmissivity was overestimated fivefold by an analyst (Figure 3). Better transmissivity estimates

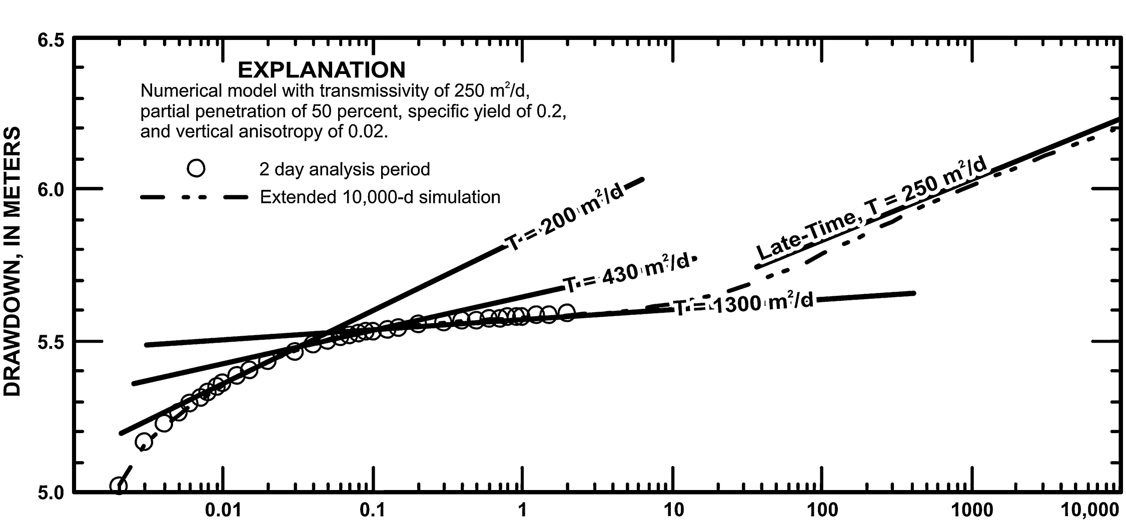

Figure 3. Analyses of an ambiguous drawdown time series where late-time response occurred after 2 d of pumping.

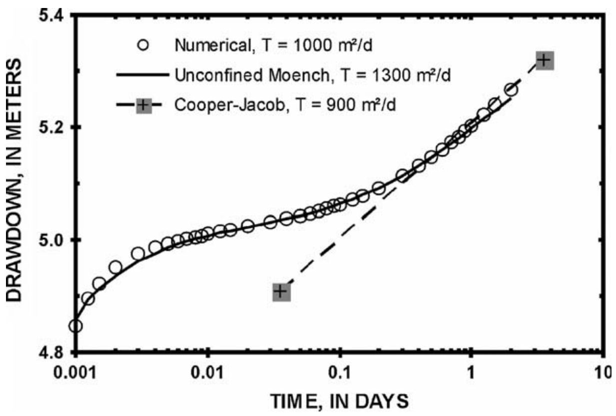

Figure 4. Example fits of unconfined Moench and Cooper-Jacob solutions to MODFLOW solution for an aquifer with a transmissivity of 1000 m^2/d, specific yield of 0.1, vertical anisotropy of 0.2, and partial penetration of 50%.

were obtained by interpreting early-time data after well-bore storage effects had dissipated. Identification of the early-time slope was ambiguous, which caused transmissivity estimates by two of the analysts to differ twofold. Late-time drawdown was not observable until 100 d of pumping had occurred. Pumping tests of 100-d duration are impractical in most situations.

Transmissivity estimates were not improved by interpreting results with an unconfined analytic solution instead of the Cooper-Jacob solution. Results from an unconfined aquifer with a transmissivity of 1000 m^2/d were interpreted with the Moench unconfined solution (Barlow and Moench 1999). Transmissivity, specific storage, vertical anisotropy, specific yield, and skin were estimated simultaneously to minimize an unweighted, sum-of-squares objective function. Transmissivity estimates ranged between 800 and 1300 m^2/d while matching all aspects of the drawdown curve (Figure 4). A similar transmissivity of 900 m^2/d was estimated by applying the Cooper-Jacob method to the late-time drawdown data.

Specific storage, vertical anisotropy, and specific yield could not be estimated uniquely with an unconfined aquifer Moench solution (Barlow and Moench 1999). Estimates of these hydraulic properties with the unconfined aquifer solution ranged over >2 orders of magnitude, and several estimates were physically unreasonable (Table 1). Initial estimates strongly affected final estimates. Initial estimates were limited to plausible values for all six solutions.

Hydraulic Conductivity

Hydraulic conductivity of confined aquifers was unambiguously determined as the transmissivity estimate divided by aquifer thickness, rather than the screen length, whenever transmissivity was 100 m^2/d or greater (Figure 2). Hydraulic conductivity was not defined clearly by either aquifer thickness or screen length where transmissivity was <50 m^2/d. Screen length might be appropriate for estimating hydraulic conductivity from transmissivity but only where transmissivity is appreciably less than 10 m^2/d.

Hydraulic conductivity of unconfined aquifers was better estimated with aquifer thickness, rather than the screen length, in >90% of the 308 aquifers with partially penetrating wells. Using aquifer thickness as the divisor gave better results for all 160 unconfined aquifers where partial penetration was 50%. Results were affected by analysts where partial penetration was 25%. For these 148 aquifers, 80% to 95% of transmissivity estimates were better interpreted with aquifer thickness. Ambiguous results occurred where transmissivity exceeded 250 m^2/d (Figure 2) and vertical anisotropy was <0.1. Hydraulic conductivity estimates ranged from 0.5 to 4 times known values where transmissivity estimates were divided by aquifer thickness. Dividing transmissivity estimates by screen lengths instead of aquifer thickness overestimated hydraulic conductivities by factors of 1.6 to 8.

Conclusions

Partial penetration and vertical anisotropy minimally affected transmissivity estimates in confined aquifers where transmissivity exceeded 50 m^2/d. Hydraulic conductivity of confined aquifers was estimated unambiguously using the transmissivity estimate divided by aquifer thickness where transmissivity exceeded 100 m^2/d.

Table 1
Six Unconfined Aquifer Moench Solutions for a Simulated Single-Well Test in an Aquifer with a Transmissivity of 1000 m^2/d, Specific Yield of 0.1, Vertical Anisotropy of 0.2, and Partial Penetration of 50%

Solution	Transmissivity (m^2/d)	Specific Storage (m^{-1})	Vertical Anisotropy	Specific Yield	Skin	Root-Mean-Square Error (m)
1	360	0.000003	0.44	0.51[1]	0.0	0.012
2	550	0.000005	0.35	0.33	3.0	0.008
3	790	0.000026	0.80[2]	0.55[1]	7.2	0.005
4	1000	0.000008	0.15	0.09	9.7	0.003
5	1300	0.0000004[3]	0.005[2]	0.002[1]	12.0	0.004
6	1400	0.000170[3]	1.40[2]	0.52[1]	17.0	0.006

[1]Specific yield is not in likely range of 0.01 to 0.35.
[2]Vertical anisotropy is not in likely range of 0.02 to 0.5.
[3]Specific storage estimate is not in likely range of 0.000003 to 0.00003 m^{-1}.

More than 90% of the unconfined aquifer transmissivities estimated by experienced analysts were within a factor of 2 of the known values. Early-time drawdowns, after wellbore storage effects had dissipated, were analyzed where transmissivity ranged between 10 and 250 m^2/d. Asymptotes to late-time drawdowns defined semilog slopes where transmissivities exceeded 500 m^2/d. Hydraulic conductivity of unconfined aquifers was better estimated with aquifer thickness for >90% of the 308 unconfined aquifers with partially penetrating wells.

Estimating hydraulic properties from a single-well pumping test with anything other than the Cooper-Jacob method is a waste of time. Transmissivity is the only hydraulic property that could be estimated uniquely. Transmissivity of an unconfined aquifer remained poorly defined if late-time data were not observed, regardless of whether the unconfined Moench or Cooper-Jacob method was used. Specific storage, vertical anisotropy, and specific yield estimates were meaningless where a single-well pumping test was analyzed with an unconfined solution.

Acknowledgments

The authors express their appreciation to Eve L. Kuniansky, Randell J. Laczniak, and David E. Prudic of the USGS who generously assisted this study by analyzing many simulated aquifer tests. The authors thank Randy Hanson, Rhett Everett, and Ed Weeks of the USGS, John Guswa, and anonymous reviewers who helped improve the paper.

References

Anderson, M.P., and W.W. Woessner. 1992. *Applied Groundwater Modeling—Simulation of Flow and Advective Transport*. San Diego, California: Academic Press Inc.

Barlow, P.M., and A.F. Moench. 1999. WTAQ—A computer program for calculating drawdowns and estimating hydraulic properties for confined and water-table aquifers. USGS Water-Resources Investigations Report 99-4225. Northborough, Massachusetts: USGS.

Belcher, W.R., P.E. Elliott, and A.L. Geldon. 2001. Hydraulic-property estimates for use with a transient ground-water flow model of the Death Valley Regional Ground-Water Flow System, Nevada and California. USGS Water-Resources Investigations Report 01-4210. Carson City, Nevada: USGS.

Clemo, T. 2002. MODFLOW-2000 for cylindrical geometry with internal flow observations and improved water table simulation. Technical Report BSU CGISS 02-01. Boise, Idaho: Boise State University.

Cooper, H.H., and C.E. Jacob. 1946. A generalized graphical method for evaluating formation constants and summarizing well field history. *American Geophysical Union Transactions* 27: 526–534.

Halford, K.J. 1997. Effects of unsaturated zone on aquifer test analysis in a shallow-aquifer system. *Ground Water* 35, no. 3: 512–522.

Hanson, R.T., and T. Nishikawa. 1996. Combined use of flowmeter and time-drawdown data to estimate hydraulic conductivities in layered aquifer systems. *Ground Water* 34, no. 1: 84–94.

Harbaugh, A.W., and M.G. McDonald. 1996. Programmer's documentation for MODFLOW-96, an update to the U.S. Geological Survey modular finite-difference ground-water flow model. USGS Open-File Report 96–486. Reston, Virginia: USGS.

McDonald, M.G., and A.W. Harbaugh. 1988. A modular three-dimensional finite-difference ground-water flow model. USGS Techniques of Water-Resources Investigations, book 6. Reston, Virginia: USGS.

Reilly, T.E., and A.W. Harbaugh. 1993. Computer note: Simulation of cylindrical flow to a well using the U.S. Geological Survey modular finite-difference ground-water flow model. *Ground Water* 31, no. 3: 489–494.

Theis, C.V. 1935. The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground water storage. *Transactions of the American Geophysical Union* 16: 519–524.